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EFFECT OF WALL COOLING ON THE MEAN STRUCTURE OF A TURBULENT BOUNDARY LAYER IN LOW-SPEED GAS FLOW†

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Abstract—The influence of wall cooling on the mean structure of a turbulent boundary layer in low-speed gas flow is discussed in terms of measured velocity and temperature profiles and friction coefficients, and comparisons are made with existing semi-empirical analyses of turbulent boundary layers. The measurements were made in an air flow through the entrance region of a smooth, isothermal tube where the free-stream velocity variation was negligible. Satisfactory agreement was found between the magnitude of the increase of the friction coefficient with cooling and values predicted from (1) a reference temperature concept, (2) Spalding and Chi's empirical correlation, and (3) Coles' transformation theory in which an appropriate value of the viscosity–temperature exponent lies between 0·7 and 1·0. Measured velocity and temperature profiles when represented in terms of u^+, T^+ and y^+ depended on a cooling parameter β, indicated by theory. Fair agreement was found between measured and predicted profiles involving Prandtl's mixing length and Coles' transformation theories.

NOMENCLATURE

c, constant;

c_p, specific heat at constant pressure;

c_f, friction coefficient,

$$\frac{c_f}{2} = \frac{\tau_w}{\rho_e u_e^2};$$

M, Mach number;

p, pressure;

Pr, Prandtl number, ν/α;

q_w, wall heat flux;

R, tube radius;

Re_θ, momentum-thickness Reynolds number, $\rho_e u_e \theta/\mu_e$;

St, Stanton number,

$$\frac{q_w}{[T_w - T_e]\,\rho_e u_e c_{p_e}};$$

T, temperature;

T^+, dimensionless temperature, equation (6);

u, velocity parallel to wall;

u_τ, friction velocity, equation (6);

u^+, dimensionless velocity, equation (6);

x, distance along wall;

y, distance normal to wall;

y^+, dimensionless normal distance, equation (6);

α, thermal diffusivity;

β, cooling parameter, equation (7);

δ, boundary-layer thickness;

δ^*, displacement thickness

$$\delta^*\left[R - \frac{\delta^*}{2}\right] = \int_0^\infty \left[1 - \frac{\rho u}{\rho_e u_e}\right][R - y]\,dy;$$

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ϵ_h, eddy diffusivity for heat;
ϵ_m, eddy diffusivity for momentum;
η, scaling function, equation (A.2);
θ, momentum thickness,

$$\theta\left[R - \frac{\theta}{2}\right] = \int_0^\infty \frac{\rho u}{\rho_e u_e}\left[1 - \frac{u}{u_e}\right][R - y]\,\mathrm{d}y;$$

κ, mixing length constant;
μ, velocity;
ν, kinematic viscosity;
ξ, scaling function, equation (A.2);
ρ, density;
σ, scaling function, equation (A.2);
τ_w, wall shear stress;
ϕ, energy thickness,

$$\phi\left[R - \frac{\phi}{2}\right] = \int_0^\infty \frac{\rho u}{\rho_e u_e}\left[1 - \left(\frac{T - T_w}{T_e - T_w}\right)\right][R - y]\,\mathrm{d}y;$$

ψ, stream function;
ω, viscosity–temperature exponent.

Subscripts and superscripts
e, condition at free-stream edge of boundary layer;
r, reference condition;
s, Coles' mean sublayer reference condition;
t, stagnation condition;
w, wall condition;
$(\bar{\ })$, constant-property value.

I. INTRODUCTION

EXPERIMENTAL investigations of turbulent-boundary-layer flow of gases over cooled surfaces have usually been associated with supersonic flow, in which frictional heating effects become important and external cooling is sometimes necessary to maintain the integrity of the surface (e.g. see the review by Spalding and Chi [1]). These are, however, numerous applications in low-speed gas flows in which temperature differences between the free stream and the surface are significant enough so that properties no longer can be considered constant. For this situation a knowledge of the influence of cooling (or heating) on the structure of a turbulent boundary layer is important. This information should also be useful as an initial step in understanding more complicated flows that involve additional effects such as compressibility and acceleration or deceleration, often found in practice.

In this experimental investigation, Pitot-tube and thermocouple measurements were made in the boundary-layer development region of turbulent flow of air through a 5 in. dia. smooth (32 microfinish) tube. Ambient air could be compressed and heated from ambient conditions to pressures of 250 psia and temperatures of 1500°R at a remote distance upstream of the tube. The 8·6 dia. long tube could be cooled to a nearly isothermal condition by 30 circumferential coolant passages. The measurements span a range of cooling conditions with wall-to-gas temperature ratios T_w/T_e extending from 1 (adiabatic wall condition) down to 0·4, a range over which apparently no boundary-layer measurements are available for turbulent boundary layers in low-speed flow. A supersonic nozzle attached to the end of the tube provided low-speed flow through the tube at a Mach number of 0·06. The free-stream velocity variation along the tube was negligible, amounting to less than 4 per cent. By varying the stagnation pressure and temperature, a relatively large range of momentum–thickness Reynolds numbers from 1500 to 36000 could be investigated at the boundary-layer measurement station near the end of the tube.

Measured velocity and temperature profiles are presented along with friction coefficients deduced either from using the Pitot tube as a Preston tube [2], or from the heat-transfer measurements made by calorimetry in the circumferential coolant passages. Semi-empirical analyses of turbulent boundary layers by Spalding and Chi [1] and Coles [3] that are based on supersonic-flow measurements are appraised by comparison to the low-speed-flow measurements made in this investigation.

II. PROBES AND MEASUREMENTS

Velocity and temperature profiles across the boundary layer were determined from simultaneous measurements of impact pressure and temperature in each of four probes located circumferentially. Three of the probes were round, 0·040 in. o.d.; the other was flattened to a smaller height of 0·014 in. and was 0·055 in. wide. The probes were moved mechanically normal to the wall by a micrometer lead screw; their location from the wall was determined with a helipot and their wall location was determined by electrical contact. The probes were motor-driven at speeds up to $\frac{1}{4}$ in./min in the outer part of the boundary layer and at slower speeds of 0·05 in./min near the wall. The pressure difference between the probe and a wall static pressure tap was measured with a pressure transducer. The output signal of the transducers and thermocouples was plotted continuously versus distance from the wall. The length and diameter of the tubes connecting the probe and wall pressure tap to the differential transducer were chosen to minimize the response of a simultaneous step-pressure input at the probe tip and wall static pressure tap. At the traversing speeds used, no difference was observed between the readings obtained by traversing from the wall to the free stream and then back again to the wall.

Thicknesses were calculated from the profile measurements by using the expressions in the nomenclature.

III. CONSTANT PROPERTIES—ADIABATIC WALL

Friction coefficients

Tests were made first with compressed air over a pressure range from 20 to 125 psia, but at ambient temperature so that there was no heat transfer to the wall. Friction coefficients were obtained for these tests by using the round boundary-layer probes as Preston tubes [2], i.e. from impact pressure measurements with the probes resting on the wall. These measurements were obtained in the law of-the-wall region, as will become evident subsequently. Patel's calibration [4] of the relationship between the wall shear stress $\bar{\tau}_w$, impact pressure $\Delta\bar{p}$, probe diameter d and gas properties was used:

$$\frac{\bar{\tau}_w d^2}{\bar{\rho}\bar{\nu}^2}\left(\frac{\Delta\bar{p}d^2}{\bar{\rho}\bar{\nu}^2}\right)^{\dagger}.$$

For later reference in the discussion of the wall-cooling results, the barred quantities refer to the

† Parentheses are used throughout the paper to indicate functional form.

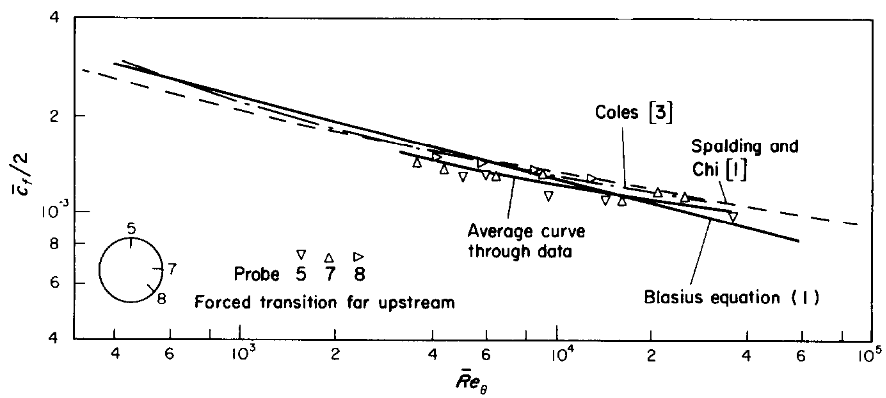

FIG. 1. Friction coefficients—constant properties.

constant-property condition. The measurements spanned values of $\Delta\bar{p}d^2/[\bar{\rho}\bar{v}^2]$ from 3×10^5 to 3×10^7, a range over which Patel's calibration of the Preston tube gives slightly higher values of the friction coefficients—5 per cent at most—than Preston's original calibration [2].

The friction coefficients so calculated are shown in Fig. 1. There is some scatter in the results obtained at three different circumferential locations 90° and 45° apart. Although some of the scatter was probably due to circumferential variation in the boundary layer found to be present, there does not appear to be any consistent trend in the results obtained from probe to probe. Consequently, some of the scatter is also due to the accuracy of the measurements, and an average curve drawn through the data points should provide a good description of the results.

Various predictions are shown in Fig. 1. The familar Blasius turbulent-boundary-layer relation is seen to lie above the data at the lower Reynolds numbers and below the data at the higher Reynolds numbers:

$$\frac{\bar{c}_f}{2} = \frac{0{\cdot}0128}{\bar{R}e_\theta^{\frac{1}{4}}}. \tag{1}$$

The power dependence is apparently too large to adequately describe the trend of the friction coefficient with Reynolds number. The trend of the results is better described by the two other constant-property relations $[\bar{c}_f/2](\bar{R}e_\theta)$ for low-speed given by Coles [3] and Spalding and Chi [1]. The latter were found by both Coles, and Spalding and Chi, to be a good representation of experimental data obtained in low-speed, constant-property flows over flat plates, and are virtually identical over the Reynolds number range of the experimental results. The experimental results of the present investigation, however, lie about 5 per cent below these predictions. The friction coefficients given by Coles in tabular form in [3] (Rand report) were obtained by expressing the boundary-layer velocity profile in terms of the law of the wall, $c + [1/\kappa] \ln \bar{y}^+$, and law of the wake [5], $w(\bar{y}/\bar{\delta})$. This profile was evaluated at the edge of the boundary layer to specify a friction law in terms of Reynolds number based on boundary-layer thickness. Then the definition of the momentum thickness was used to convert the results to a friction-coefficient dependence on momentum thickness Reynolds number. Spalding and Chi's relation, given in tabular form in [1], was obtained principally in the same way, but the velocity profile was expressed in the form $\bar{y}^+(\bar{u}^+)$, which allowed a direct evaluation of the integral expression for

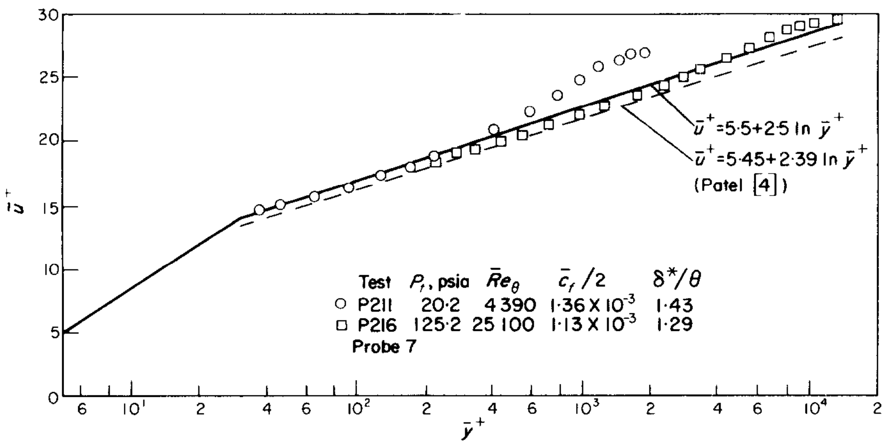

FIG. 2. Velocity profiles—constant properties.

the momentum thickness to yield a friction relation in the inverse form, i.e. $\overline{Re}_\theta(\bar{c}_f/2)$. The empirical expression for the velocity profile described by Spalding [6] exhibits the behaviour that $\bar{u}^+ \rightarrow \bar{y}^+$ in the laminar sublayer and that $\bar{u} \rightarrow c + [1/\kappa] \ln \bar{y}^+$, the law of the wall, farther away from the wall. Including the wake function or not apparently makes little difference in the predicted friction-coefficients relation over a relatively large range of Reynolds numbers, there being significant differences only at low Reynolds numbers.

Boundary-layer profiles

Velocity profiles for adiabatic wall operation are shown in Fig. 2 in terms of $\bar{u}^+$ and $\bar{y}^+$. To clearly indicate the nature of the profiles, only two of the numerous profiles that were obtained are shown. The profiles agree well with the trend indicated by either form of the law of the wall shown as

$$\bar{u}^+ = c + \frac{1}{\kappa} \ln \bar{y}^+. \tag{2}$$

Better agreement in magnitude, however, is provided by the form given by Patel that is compatible with his Preston-tube measurements. A tube displacement effect is noticeable in the wall vicinity; for example, at the lower Reynolds number, velocities appear to correspond to effective tube locations farther away from the wall; however, no correction was made for this effect, which is important only in the immediate vicinity of the wall. In the outer part of the boundary layer, the wakelike behaviour found in turbulent boundary layers is evident (e.g. see Coles [5]). The other velocity profiles obtained for an adiabatic wall are similar in shape to those shown in Fig. 2, displaying law-of-the-wall and wakelike regions.

IV. COOLED WALL

Friction coefficients

With the structure of the boundary layer known for adiabatic wall operation—the results being typical of those found in investigations of turbulent boundary layers in constant-property, low-speed flow—the compressed air was heated upstream and the tube wall was cooled. The tube wall was maintained at a nearly isothermal condition by the separate circumferential coolant passages, and the local heat transfer from the gas to the wall was measured along the tube [7]. These heat-transfer data are believed to be accurate to about ± 5 per cent when the gas temperature was 1500°R (wall-to-gas-temperature ratio T_w/T_e from about 0·4 to 0·5). At a lower gas temperature of 1000°R (T_w/T_e of about 0·6) the heat-transfer data are less accurate. An attempt to obtain heat-transfer data with a small difference between free-stream and wall-temperature (T_w/T_e near 1) did not yield any useful results primarily because of the small water-temperature rise in the coolant passages, which was difficult to measure accurately, and also because of the uncertainty in the actual gas-side wall temperatures that were obtained from the measured wall heat fluxes and thermocouple measurements on the coolant-side wall. Whereas knowledge of the gas-side wall temperature to within about 10°F leads to an insignificant error at higher gas temperatures, this uncertainty becomes important when the difference between the gas and wall temperature is not large.

Experimental data were obtained at two conditions: one in which natural transition from a laminar to a turbulent boundary layer occurred along the tube upstream of the boundary-layer probe station; the other for forced transition produced by a small trip ring located far upstream at the tube inlet, this latter condition being the same for which the constant-property results were obtained. From the boundary-layer measurements, thicknesses were calculated that permit an overall view of the flow and have a bearing on the subsequent discussion.

The effect of wall cooling, for example for forced transition, was to reduce the displacement thickness relative to the momentum thickness; values of δ^*/θ were between 0·45 and 0·7, whereas constant-property values ranged from 1·28 to 1·46. This trend is in agreement with the analysis of [8]. Compared to the tube radius, the displacement thickness was small, amounting to less than 4 per cent. Values of the ratio of the energy thickness to the momentum thickness ϕ/θ were between 1·0 and 1·3. The ratio of thermal to velocity boundary-layer thickness δ_t/δ should be essentially the same as ϕ/θ for air ($Pr = 0{\cdot}7$) if both the velocity and temperature boundary layers originated in the same vicinity, the tube wall were isothermal, and the free-stream velocity variation were negligible.

These latter conditions were nearly realized for the flow through the cooled tube, and formed the basis for calculating friction coefficients from the local heat-transfer measurements by the following form of Reynolds analogy relating the Stanton number to the friction coefficient

$$\frac{St}{c_f/2} = 1{\cdot}16, \qquad \text{for air, } Pr = 0{\cdot}7. \tag{3}$$

The particular relation used was based on the careful heat-transfer measurements by Reynolds *et al.* [9] in a low-speed, essentially constant-property, turbulent-boundary-layer air flow over a flat, isothermal plate. The friction coefficient was obtained by Reynolds *et al.* from the Schultz–Grunow relation (e.g. see Schlichting [10], p. 600), a relation apparently in good agreement with measurements on a flat plate. Recent flat-plate heat-transfer measurements by Chi and Spalding [11] in a low-speed air flow with wall-cooling support the use of equation (3). This relation is nearly identical to the von Karman analogy.

$$\frac{St}{c_f/2} = \frac{1}{1 - 3[c_f/2]^{\frac{1}{2}}}, \qquad \text{for air, } Pr = 0{\cdot}7$$

in the region of interest in this investigation and in the investigations by Reynolds *et al.* and Chi and Spalding. It should be remarked that if the Colburn equation

$$\frac{St}{c_f/2} = \frac{1}{Pr^{\frac{2}{3}}} = 1{\cdot}27, \qquad \text{for air, } Pr = 0{\cdot}7$$

were used, calculated values of the friction coefficient would be about 10 per cent lower than values obtained from equation (3). Of note is that in a laminar boundary layer in a variable-property, constant free-stream velocity, low-speed flow over a cooled, isothermal wall, the Colburn equation provides a good approximation of the actual relationship between heat transfer and friction, as can be seen by comparing the values in Table 1 from the calculations by Back [12] to the value of $St/[c_f/2] = 1{\cdot}27$. However, in a turbulent boundary layer the factor $St/[c_f/2]$ may be less, because the eddy diffusivities for momentum and heat transfer, ϵ_m and ϵ_h, respectively, may be nearer to each other than the molecular diffusivities, i.e. $Pr = \nu/\alpha = 0{\cdot}7$ for air. Consequently, although simultaneous measurements of heat flux and shear stress at the wall has not been made, the use of equation (3) for the conditions of this investigation appears to be plausible and yields semilocal friction coefficients averaged around the circumference of the tube. Subsequent discussion will shed further light on the accuracy of the friction coefficients when they are considered in conjunction with the boundary-layer measurements.

Friction coefficients shown in Fig. 3 exhibit the effect of cooling, the degree of cooling being indicated by the wall-to-free-stream temperature ratio T_w/T_e. The properties in both the friction coefficient and momentum–thickness Reynolds number are evaluated at the free-stream condition so that the effect of cooling resides in the results at the different values of T_w/T_e. At a given momentum-thickness Reynolds number the effect of wall cooling, i.e. decreasing T_w/T_e, is to increase the friction coefficient

Table 1. Effect of cooling on the Reynolds analogy factor for a laminar boundary layer in a variable-property, constant free-stream velocity, low-speed flow over a cooled, isothermal wall;

$Pr = 0{\cdot}7, \quad \omega = 0{\cdot}7, \quad c_p = \text{const}$

(*Back* [12])

T_w/T_e	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{2}{3}$	1·0
$St/[c_f/2]$	1·254	1·256	1·258	1·260

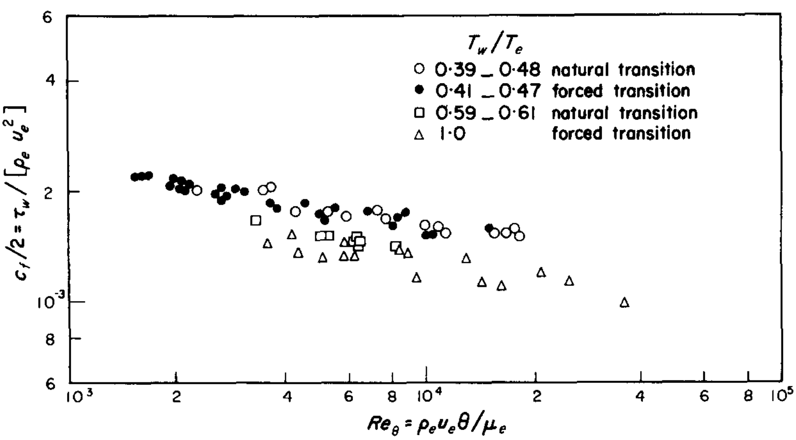

FIG. 3. Friction coefficients with and without cooling.

above the constant-property values that are also shown in Fig. 3. Friction coefficients obtained for both natural and forced transition upstream agree with each other, as indicated by the results for T_w/T_e from 0·39 to 0·48.

Although the magnitude of the increase of the friction coefficient with cooling can be determined directly from Fig. 3, it is useful to view the results as some function of $c_f/2$ and Re_θ vs. T_w/T_e to directly observed the effect of cooling. The friction group $[c_f/2][Re_\theta]^{1/5}$ was chosen for this purpose, not because this group necessarily provides a correlation of the results over the range of T_w/T_e of interest, but because of various simple functions that could be chosen—e.g. $[c_f/2][Re_\theta]^{1/4}$—the results in terms of the group $[c_f/2][Re_\theta]^{1/5}$ spread less with Re_θ at a particular value of T_w/T_e.

Representation of the data in terms of the friction group $[c_f/2][Re_\theta]^{1/5}$ vs. T_w/T_e is shown

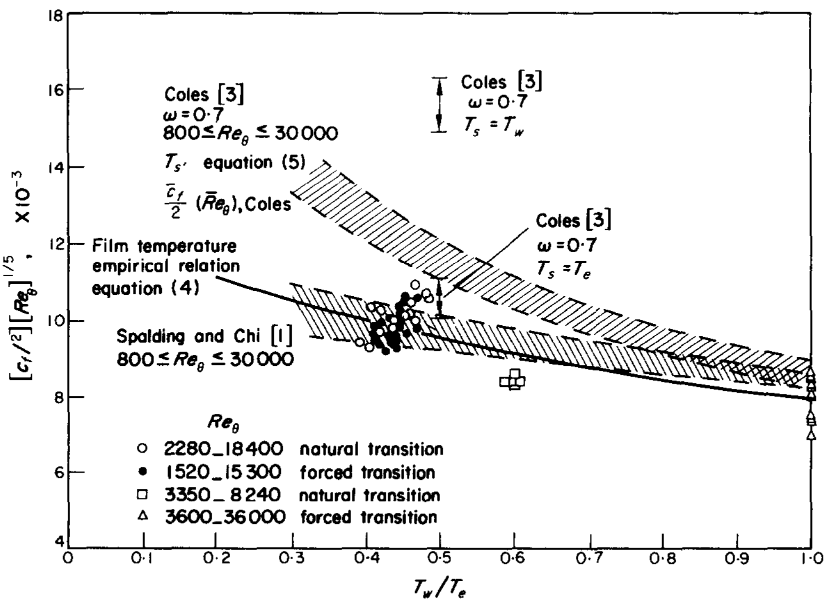

FIG. 4. Effect of cooling on the friction group.

in Fig. 4 to indicate the effect of wall cooling. The increase of the friction group above the constant-property values with cooling amounts to about 20 per cent at a ratio of wall-to-free-stream temperature of about 0·5. Various predictions are also shown in Fig. 4 and some discussion pertaining to these predictions is forthcoming.

Predicted friction coefficients

Perhaps the simplest approach to the variable-property problem, but certainly the one that provides the least insight, is to modify a constant-property relation by evaluation of properties ρ and μ at a reference temperature. Application of this concept to the relation

$$\frac{\tau_w}{\rho_r u_e^2}\left[\frac{\rho_r u_e \theta}{\mu_r}\right]^{\frac{1}{4}} = A$$

yields the following for the friction group in the variable-property flow

$$\frac{c_f}{2}[Re_\theta]^{\frac{1}{4}} = A\left[\frac{\rho_r}{\rho_e}\right]^{\frac{3}{4}}\left[\frac{\mu_r}{\mu_e}\right]^{\frac{1}{4}}.$$

Prescription of a viscosity–temperature relation and a reference temperature then allows an appraisal of the prediction by comparison with the data, the constant A being chosen to agree with the constant-property results. Over the temperature range of interest, i.e. ambient temperature to 1500°R, a good approximation of the actual variation of viscosity of air with temperature is given by the simple power relation $\mu \propto T^\omega$, with $\omega = 0{\cdot}7$. Choosing a reference temperature at the free-stream value would yield a friction group that would be invariable with cooling, a relation that would lie below the experimental results. On the other hand, a choice of the wall temperature as a reference temperature would result in too large an increase in the predicted friction group with cooling. An appropriate reference temperature should lie somewhere between the wall and the free-stream value, and the choice of a temperature halfway between the wall and the free-stream temperature, often referred to as the film temperature $T_f = [T_w + T_e]/2$, provides good agreement with the results as shown in Fig. 4:

$$\frac{c_f}{2}[Re_\theta]^{\frac{1}{4}} = A\left[\frac{2}{1 + T_w/T_e}\right]^{(\frac{1}{4})(4-\omega)}, \quad A = 7{\cdot}9 \times 10^{-3}, \quad \omega = 0{\cdot}7. \qquad (4)$$

Spalding and Chi

Another prediction shown in Fig. 4 is from the empirical method of Spalding and Chi [1] determined from supersonic-flow measurements on flat plates with and without heat transfer. This method is briefly described in the Appendix and will be subsequently discussed in connection with Coles' transformation theory. The prediction from the empirical method of Spalding and Chi is in good agreement with the low-speed-flow experimental results of this investigation as shown in Fig. 4. This finding is also in agreement with the flat-plate heat-transfer measurements that were later made by Chi and Spalding [11] in a low-speed air flow with cooling (T_w/T_e extending down to 0·37) where, in the absence of boundary-layer measurements, the data were correlated on the basis of a Reynolds number containing the length along the plate. Subsequent numerical solutions of the equations for a turbulent boundary layer, including compressibility and wall cooling and heating, for flat-plate variable-property flow have been made by Patankar [13] using Prandtl's mixing-length theory and assuming the eddy diffusivities for momentum and heat transfer ϵ_m/ϵ_h to be 0·9. These predictions were found to agree well with the empirical method of Spalding and Chi and the low-speed-flow heat-transfer measurements by Chi and Spalding. We shall later return to the mixing-length concept in the discussion of the boundary-layer velocity and temperature profiles.

Coles' transformation theory

The prediction methld of Coles [3] is based on the concept of transforming the equations for a steady, compressible boundary layer to incompressible form. A brief description of the transformation that is relevant to the subsequent discussion is given in the Appendix. Coles' ideas are applied herein to two low-speed flows with constant free-stream velocity, one a variable-property flow with heat transfer and the other a constant-property flow also with heat transfer, but in which temperature differences are small so that the properties are essentially constant. Before going into the structure of the boundary layer implied by the transformation, which will be subsequently discussed, the results of the transformation theory related to the friction relation given by equation (A.3) are compared with the experimental results that have been presented. For low-speed flow Coles' relation for the mean sublayer temperature defined in equation (A.6) and evaluated from supersonic-flow data reduces to

$$\frac{T_s}{T_w} = 1 - \langle f \rangle \left[1 - \frac{T_e}{T_w}\right]\left[\frac{\bar{c}_f}{2}\right]^{\frac{1}{2}} \quad (5)$$

which implies that the Prandtl number is unity for a flow with heat transfer. The function $\langle f \rangle$ defined by

$$\langle f \rangle = \frac{1}{\bar{y}_s^+}\int_0^{\bar{y}_s^+} \bar{u}^+ \, \mathrm{d}\bar{y}^+$$

was found by Coles to be 17·2. This value implies a relatively thick sublayer extending to $\bar{y}_s^+ = 430$, a location well within the turbulent region of a turbulent boundary layer in a constant-property low-speed flow.

Using the tabular values given by Coles [3] (Rand report) for the constant-property friction relation $[\bar{c}_f/2]\,(\overline{Re}_\theta)$ shown in Fig. 2, the variable-property friction relation is obtained from equations (5), (A.3) and (A.5) once a viscosity–temperature relationship is specified. For purposes herein, the power relation $\mu \propto T^{\omega}$ is considered for which the friction relation can be written in the following form for a variable-property flow

$$\frac{c_f}{2}\left(Re_\theta, \frac{T_w}{T_e}, \omega; \bar{c}_f(\overline{Re}_\theta), \bar{y}_s^+\right).$$

Selection of a value of ω of 0·7 then yields values of the friction group $[c_f/2][Re_\theta]^{\frac{1}{4}}$ shown in Fig. 4. The predicted effect of cooling on the friction group is apparently larger than the observed increase. The calculated sublayer temperatures were closer to the free-stream temperature than the wall temperature. An indication of this behaviour is shown in particular at $T_w/T_e = 0{\cdot}5$, where instead of evaluating the sublayer temperature from equation (5), it was assumed equal first to the wall and then to the free-stream temperature. Apparently, if any correspondence is to be achieved with the data by using a viscosity relation for air consistent with the conditions of the measurements, the mean sublayer temperature would need to be evaluated near, if not equal to, the free-stream temperature. The notion that virtually the entire boundary layer is the sublayer is hardly acceptable, and consequently, other implications about the transformation with regard to the viscosity–temperature relation should be examined.

Since the relations between the two flows given by equation (A.3) leading to Coles' "law" of of corresponding stations (equation A.4) apply to both laminar and turbulent boundary layers, some information can be obtained from laminar boundary layers. The difference between exact calculations for a laminar boundary layer in a variable-property, constant free-stream velocity, low-speed flow over a cooled, isothermal wall, and the prediction from the law of corresponding stations is shown in Fig. 5 in terms of the group

$$\frac{c_f\, Re_\theta}{\bar{c}_f\, \overline{Re}_\theta}.$$

Predictions from the law of corresponding stations (equation A.4) with $\omega = 0{\cdot}7$ exceed the actual values. This trend is in the same direction as was indicated by comparison with the

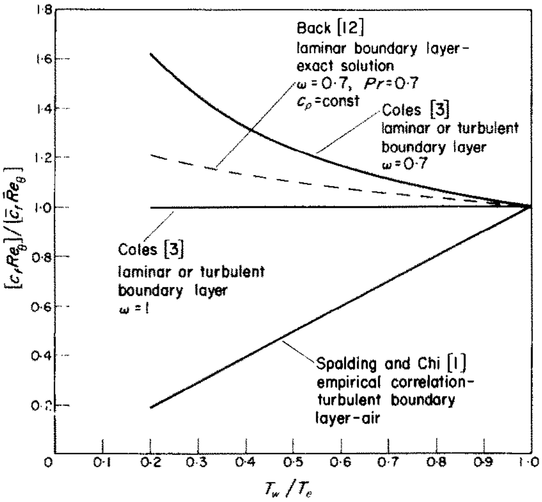

FIG. 5. Effect of cooling on Coles' law of corresponding stations in low-speed, laminar and turbulent boundary layers at constant free-stream velocity.

experimental results shown in Fig. 4 for a turbulent boundary layer. If the viscosity is taken to be proportional to temperature, i.e. $\omega = 1$, the law of corresponding stations (equation A.4) becomes

$$\frac{c_f}{2} Re_\theta = \frac{\bar{c}_f}{2} \bar{R}e_\theta$$

where the group $[c_f/2]Re_\theta$ is invariable with cooling. For this choice the prediction lies below the exact values.

Application of the law of corresponding stations to laminar boundary layers indicates the difficulty that might be expected for turbulent boundary layers when the viscosity of a gas is not proportional to temperature, as is the case for most gases at moderate temperatures. From the comparison of data for a turbulent boundary layer shown in Fig. 4 and the trend of the predictions for a laminar boundary layer shown in Fig. 5, it appears that to achieve a correspondence between prediction and experiment, a fictitious viscosity relation needs to be invoked in which the exponent ω for a power relation would lie somewhere between 0·7 and 1. Lewis [14] subsequently has suggested that the condition $\rho\mu = $ const. across the boundary layer, i.e. $\mu \propto T$, $\omega = 1$ for a perfect gas, is also a requirement in Coles' transformation [3] when applied to a flow with heat transfer.

The choice of a viscosity-temperature law in the prediction from Coles' theory is demonstrated in Fig. 6 for a turbulent boundary layer. In the upper part of Fig. 6, where the transformation was applied to the constant property friction relation $[\bar{c}_f/2](\overline{Re}_\theta)$ given by Coles, good agreement with the experimental results is found for $\omega = 1$. However, it is more appropriate to apply the transformation on a purely experimental basis, i.e. to the measured friction coefficients with constant properties. The predictions so obtained are shown in the lower part of Fig. 6. They indicate that an appropriate value of the exponent ω lies somewhat between 0·7 and 1, and this is consistent with that observed from the law of corresponding stations.

Before discussing the boundary-layer profiles, mention should be made of Spalding and Chi's [1] empirical method when it is expressed in the form of the law of corresponding stations given by equation (A.1) and shown in Fig. 5. Although the predicted friction relation is in good agreement with the experimental results for a low-speed flow shown in Fig. 4, the friction group $[c_f/2]Re_\theta$ varies with cooling in an opposite way from Coles' law of corresponding stations. The implication of this behaviour is not clear, in particular if the Spalding and Chi method is applied outside the range of conditions on which it is based.

Boundary-layer profiles

Some of the velocity and temperature profiles obtained for wall cooling are shown in Fig. 7 in terms of u^+, T^+ and y^+, defined as

$$u^+ = \frac{u}{u_\tau}, \qquad T^+ = \frac{T_w - T}{q_w/\rho_w u_\tau c_{p_w}}, \qquad y^+ = \frac{\rho_w u_\tau y}{\mu_w} \tag{6}$$

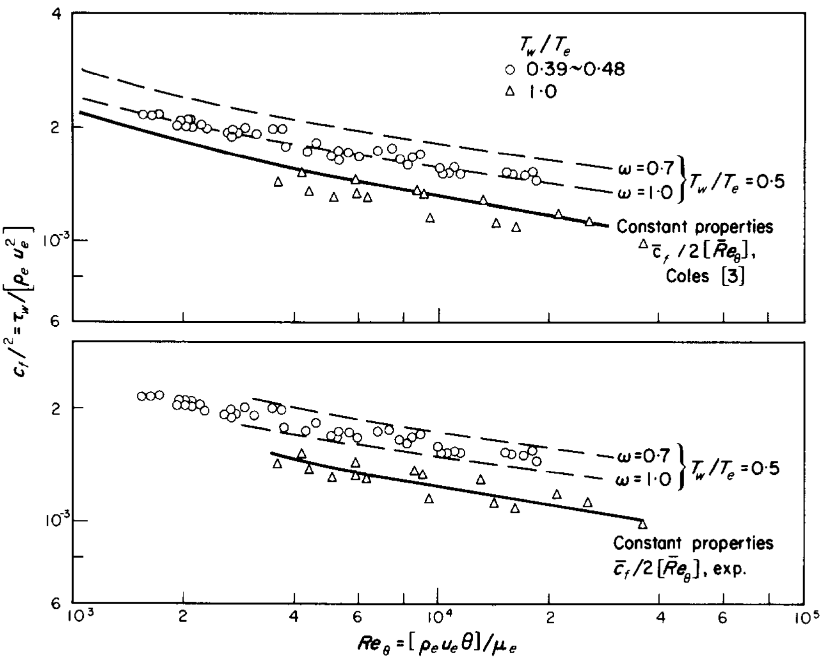

FIG. 6. Comparison between friction coefficients and Coles' transformation theory.

where

$$u_\tau = \left[\frac{\tau_w}{\rho_w}\right]^{\frac{1}{2}}.$$

These profiles were obtained with the small, flattened probe. Since the wall temperature is known and the wall heat flux was measured, the measured gas temperatures in the immediate vicinity of the wall could be extrapolated toward the wall. However, when this extrapolation was made, the temperature readings with the probe resting on the wall and at distances of 2–3 mils from the wall were found to be somewhat low, apparently because of heat convection to the cooled wall from the thermocouple surface adjacent to the wall. Consequently, these temperatures were adjusted to be consistent with extrapolation to the known temperature gradient at the wall. Errors associated with this extrapolation are expected to be small, especially for the velocity profile, since velocity depends on the square root of the density or temperature measurement. As an overall check on the profiles, values of the energy thickness calculated from the profiles were in good agreement with values associated with the energy defect in the flow as a consequence of heat transfer to the upstream tube wall.

The measured profiles in Fig. 7 generally lie above the constant-property reference profiles. The amount of departure is dependent upon a parameter β (see Deissler [15]) that will be referred to as the cooling parameter ($\beta < 0$), which is defined as

$$\beta = \frac{q_w}{T_w \rho_w u_\tau c_{P_w}}. \tag{7}$$

In the outer part of the layer the wakelike behaviour observed for adiabatic wall operation is again evident, not only in the velocity profiles but also in the temperature profiles. Other measured velocity and temperature profiles that were obtained display the same features as the profiles shown in Fig. 7. The circum-

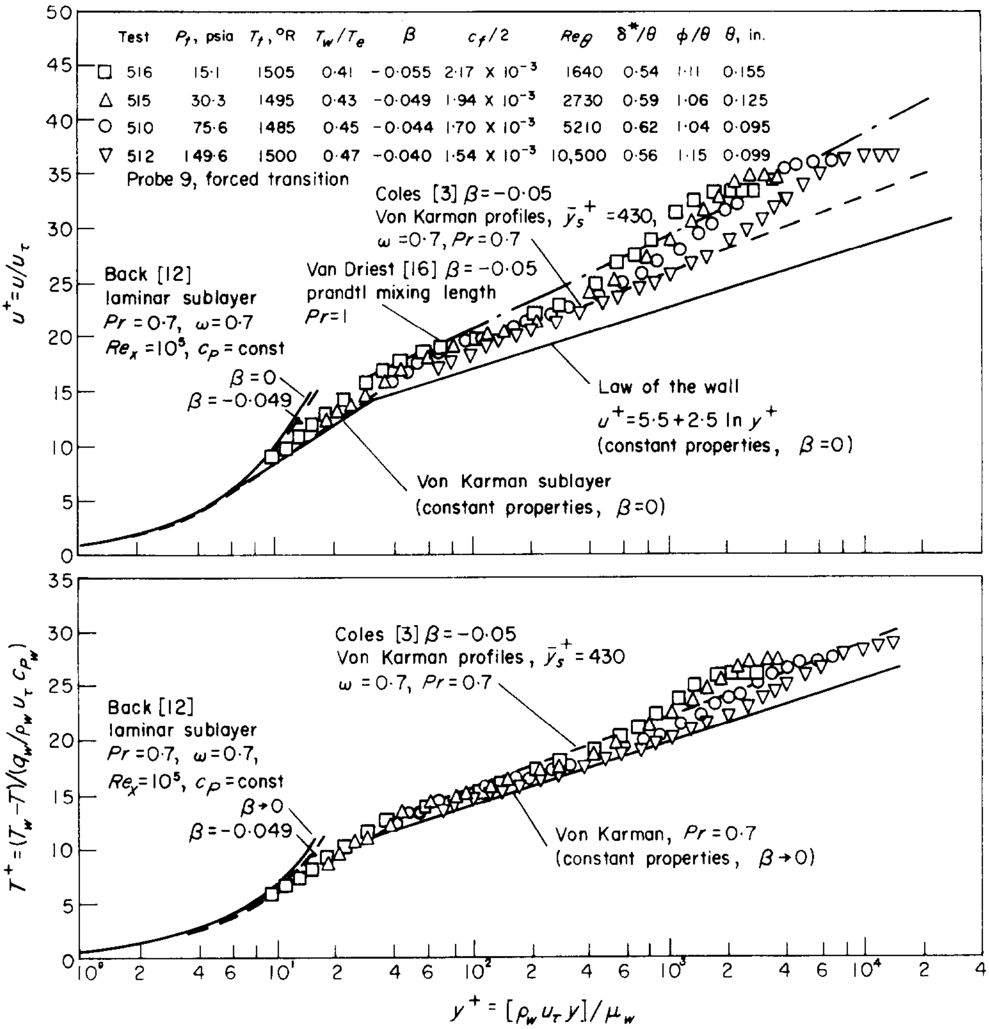

FIG. 7. Velocity and temperature profiles with cooling.

ferential variation found with adiabatic wall operation was apparently reduced with cooling, since the probe readings at the different circumferential locations were found to be in better agreement with each other.

Predicted velocity and temperature profiles are shown in Fig. 7 to indicate the effect of cooling on the structure of a turbulent boundary layer in low-speed flow. These are subsequently discussed in light of the measured profiles.

Mixing length theory

Perhaps the simplest appraisal of the effect of cooling on the turbulent portion of the boundary layer would be to extend Prandtl's mixing-length theory ($l = \kappa y$) to a variable-property flow. The prediction by Van Driest [16], who expressed the density variation in terms of velocity

$$\frac{\rho}{\rho_w} = \frac{T_w}{T} = \frac{1}{1 - \beta u^+}$$

by assuming a linear variation between temperature and velocity $T^+ = u^+$, ($Pr = 1$, $\epsilon_m = \epsilon_h$), takes on the following form for a low-speed flow

$$[1 - \beta u^+]^{\frac{1}{2}} - 1 = -\frac{\beta}{2}\left[c + \frac{1}{\kappa}\ln y^+\right]. \quad (8)$$

Specification of the values of c and κ, herein taken as 5·5 and 0·4, respectively, indicates the explicit dependence of the velocity profiles on the cooling parameter $u^+(y^+, \beta)$. The profile shown for $\beta = -0{\cdot}05$ is in agreement with the measured velocity profiles at nearly the same value of β in the initial part of the law-of-the-wall region; however, the predicted profile slope in the law-of-the-wall region is larger than indicated by the measured profiles. In the outer portion of the boundary layer, where the wakeline behaviour is observable, the prediction lies below the data, a situation also found for constant-property flow. However, on an overall basis, the measured profile is well represented by the prediction based on the mixing-length concept, and this correspondence is consistent with that implied by the Spalding and Chi correlation [1].

Coles' transformation theory

To gain some idea of how cooling might affect the sublayer as well as the turbulent region in a variable-property flow, Coles' transformation [3] is applied to the constant-property profiles as follows

$$\bar{u}^+ = f\left(\frac{\bar{\rho}\bar{u}_\tau \bar{y}}{\bar{\mu}}\right)$$

$$\bar{T}^+ = g\left(\frac{\bar{\rho}\bar{u}_\tau \bar{y}}{\bar{\mu}}, \bar{P}r\right)$$

where

$$\bar{u}^+ = \frac{\bar{u}}{\bar{u}_\tau}, \qquad \bar{T}^+ = \frac{\bar{T}_w - \bar{T}}{\bar{q}_w/\bar{\rho}\bar{u}_\tau \bar{c}_p},$$

$$\text{and} \qquad \bar{u}_\tau = \left[\frac{\bar{\tau}_w}{\bar{\rho}}\right]^{\frac{1}{2}}. \quad (9)$$

With the use of Crocco's specification [17] that the temperature ratio is the same in the two flows, i.e. $\bar{T}/\bar{T}_e = T/T_e$, from which it follows that $(T - T_w)/(T_e - T_w) = (\bar{T} - \bar{T}_w)/(\bar{T}_e - \bar{T}_w)$ (the small variation in the specific heat is neglected herein), similar to the velocity ratio $\bar{u}/\bar{u}_e = u/u_e$ from Coles' transformation, and that the Prandtl number is the same in the two flows, the constant-property relations take on the following form for a variable-property flow

$$\left[\frac{\mu_w}{\mu_s}\right]^{\frac{1}{2}} u^+ = f\left(\left[\frac{\mu_w}{\mu_s}\right]^{\frac{1}{2}} \frac{u_\tau}{\mu_w}\int_0^y \rho \,\mathrm{d}y\right)$$

$$\left[\frac{\mu_w}{\mu_s}\right]^{\frac{1}{2}} T^+ = g\left(\left[\left[\frac{\mu_w}{\mu_s}\right]^{\frac{1}{2}} \frac{u_\tau}{\mu_w}\int_0^y \rho \,\mathrm{d}y\right], Pr\right)$$

where u^+, T^+ and u_τ are defined as before· The profiles can be evaluated either in the variable-property coordinates u^+, T^+ and y^+ or the constant-property coordinates $\bar{u}^+$, $\bar{T}^+$ and $\bar{y}^+$ as follows

$$u^+ = \left[\frac{\mu_s}{\mu_w}\right]^{\frac{1}{2}} \bar{u}^+$$

$$T^+ = \left[\frac{\mu_s}{\mu_w}\right]^{\frac{1}{2}} \bar{T}^+ \quad (10)$$

$$y^+ = \left[\frac{\mu_s}{\mu_w}\right]^{\frac{1}{2}} \left[\bar{y}^+ - \beta\left[\frac{\mu_s}{\mu_w}\right]^{\frac{1}{2}} \int_0^{\bar{y}^+} \bar{T}^+ \,\mathrm{d}\bar{y}^+\right]$$

or

$$\bar{y}^+ = \left[\frac{\mu_w}{\mu_s}\right]^{\frac{1}{2}} \frac{u_\tau}{\mu_w}\int_0^y \rho \,\mathrm{d}y$$

$$= \left[\frac{\mu_w}{\mu_s}\right]^{\frac{1}{2}} \int_0^{y^+} \frac{1}{1 - \beta T^+}\,\mathrm{d}y^+.$$

The y^+ to $\bar{y}^+$ relation was obtained by inverting the $\bar{y}$ to y transformation. The cooling parameter β arises when the density is expressed in terms of temperature in the form

$$\frac{\rho}{\rho_w} = \frac{T_w}{T} = \frac{1}{1 - \beta T^+}. \tag{11}$$

The mean sublayer temperature defined by Coles' equation (A.6) can be written in the form

$$\frac{T_s}{T_w} = 1 - \beta \left[\frac{\mu_s}{\mu_w}\right]^{\frac{1}{2}} \left[\frac{1}{\bar{y}_s^+} \int_0^{\bar{y}_s^+} \bar{T}^+ \, \mathrm{d}\bar{y}^+\right]. \tag{12}$$

In particular for a gas with Prandtl number of unity and equal eddy diffusivities for momentum and heat transfer, i.e. $\bar{T}^+ = \bar{u}^+$, equation (12) reduces to that given by Coles' equation (5), which can be seen by evaluating equation (11) at the edge of the boundary layer to express β in terms of T_w/T_e and $c_f/2$, noting that $T_e^+ = u_e^+$, and then by expressing $c_f/2$ in terms of $\bar{c}_f/2$ by equation (A.3).

It should be noted that even for a gas with Prandtl number less than unity, as in the case for most gases, the mean sublayer temperature given by equation (5) hardly differs from that obtained from the more general relation for any Prandtl number

$$\frac{T_s}{T_w} = 1 - \left[\frac{St}{c_f/2}\right]\left[\frac{1}{\bar{y}_s^+} \int_0^{\bar{y}_s^+} \bar{T}^+ \, \mathrm{d}\bar{y}^+\right] \times \left[1 - \frac{T_e}{T_w}\right]\left[\frac{\bar{c}_f}{2}\right]^{\frac{1}{2}}.$$

For example, for $Pr = 0{\cdot}7$, the product of the first two terms obtained by using the constant property von Kármán temperature profile and $\bar{y}_s^+ = 430$ is as follows

$$\left[\frac{St}{c_f/2}\right]\left[\frac{1}{\bar{y}_s^+} \int_0^{\bar{y}_s^+} \bar{T}^+ \, \mathrm{d}\bar{y}^+\right] = 1{\cdot}16\,[15{\cdot}1] = 17{\cdot}5.$$

Since this value is essentially the same as Coles' value of $\langle f \rangle = 17{\cdot}2$, the specification of the friction coefficient from Coles' transformation theory for $Pr = 1$ would scarcely differ from that for most gases with Prandtl number less than unity.

As a direct appraisal of the transformation theory, velocity profiles obtained with wall cooling can be transformed to the constant-property coordinates and compared directly to the corresponding constant-property profiles on a purely experimental basis. This comparison is shown in Fig. 8 for two operating conditions where the variable-property profiles extend well into the viscous sublayer. The transformed variable-property velocity profiles in which the actual variation of viscosity with temperature was used, i.e. $\omega = 0{\cdot}7$, along with the constant-property von Kármán temperature profile and $\bar{y}_s^+ = 430$, are seen to lie slightly above the measured constant-property profiles in the law-of-the-wall region where the profiles overlap one another. If, however, the variable-property velocity profiles were transformed by taking viscosity proportional to temperature, the transformed velocity profiles as shown by the dashed curves in Fig. 8 would lie below the measured constant-property profiles. Consequently, a direct comparison of the measured velocity profiles indicates that an appropriate value of the exponent ω would again lie somewhere between 0·7 and 1, and this is consistent with that observed from a comparison of the measured friction coefficients and from the law of corresponding stations. This overall correspondence also establishes confidence in the values of the friction coefficient obtained from the heat-transfer measurements for the cooled-wall flow, since the wall shear stress was not measured directly.

It is also useful to view the effect of cooling that is indicated by the predicted velocity and temperature profiles from transformation theory in the variable-property flow. The relations given by equations (10) and (12) can be expressed in the following form

$$u^+[y^+, \beta, \omega; f(\bar{y}^+), g(\bar{y}^+, Pr), \bar{y}_s^+]$$
$$T^+[y^+, \beta, \omega; g(\bar{y}^+, Pr), \bar{y}_s^+].$$

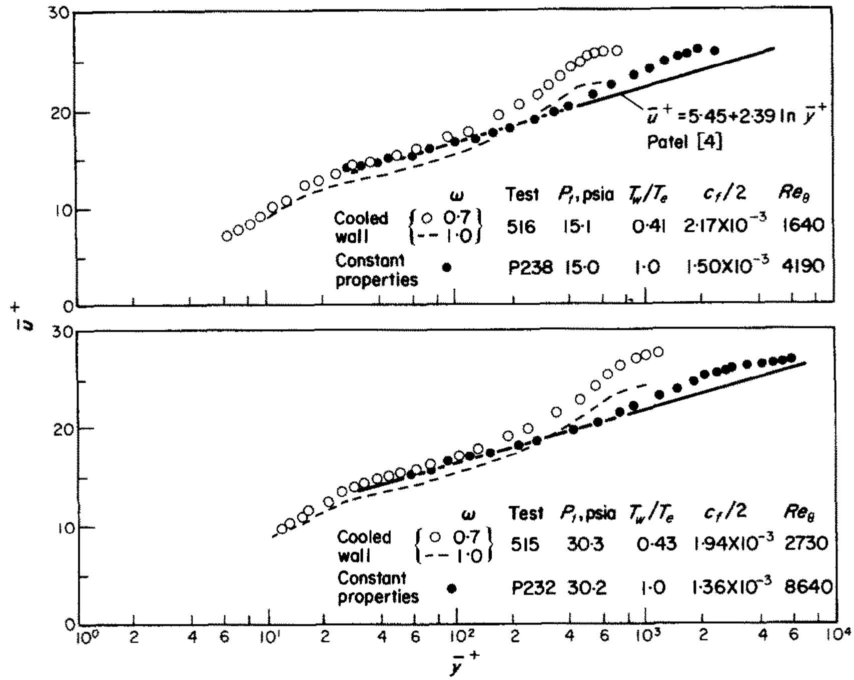

FIG. 8. Velocity profiles in constant-property coordinates.

To indicate the trend of the prediction, a calculation was carried out for $\beta = -0{\cdot}05$, $\omega = 0{\cdot}7$, $\bar{y}_s^+ = 430$ (corresponding to Coles' sublayer value), and the constant-property von Kármán profiles f and g for $Pr = 0{\cdot}7$ (Fig. 7). Before discussing the prediction that is shown in Fig. 7, it should be pointed out that the predicted profiles would be shifted downward if a more appropriate constant-property, law-of-the-wall relation were used (Fig. 2) and upward if ω were chosen as 1, so that the behavior observed in the constant-property coordinates (Fig. 8) are consistent with those in the variable-property coordinates, as indeed must be the case. In the laminar sublayer the predicted profiles with cooling lie below the constant-property relation, and this trend is similar to that expected if the boundary layer were wholly laminar. In the region where both molecular and turbulent transport are important, referred to as the buffer layer, the predicted profiles cross over and then lie above the constant-property profiles, and are in good agreement with the measured velocity and temperature profiles. With cooling there is a predicted increase in the value of y^+ associated with the constant-property sublayer, and the data appear to support this trend. In the turbulent region the velocity profile is near the profile deduced from mixing-length theory, but the profile slope is less, in better agreement with the measured profiles in the law-of-the-wall region. In the outer portion of the boundary layer, better correspondence with the measured velocity profiles would probably be achieved if the constant-property profile $f(\bar{y}^+)$ included an appropriate wake function $w(\bar{y}/\bar{\delta})$. It should be noted that the predicted velocity profile would lie below that profile shown if the sublayer thickness were assumed to be less, i.e. smaller $\bar{y}_s^+$. In this regard, for the calculation, the mean sublayer temperature is still closer to the freestream than the wall temperature, as can be observed by comparing the value of $T_s/T_w = 1{\cdot}96$ to the measured value of $T_e/T_w = 2{\cdot}4$, corresponding to the tests at $\beta \cong -0{\cdot}05$.

Whereas the effect of Prandtl number on the predicted velocity profile is small—the profile for an assumed Prandtl number of 1 would lie slightly above the profile shown in Fig. 7 for $Pr = 0{\cdot}7$—there is an effect of Prandtl number on the temperature profile. The predicted temperature profile for $Pr = 1$ would lie above the profile for $Pr = 0{\cdot}7$, since it would be identical to the velocity profile if the eddy diffusivities for the momentum and heat transfer were the same and the wall is isothermal. For this case the Crocco relation

$$\frac{T - T_w}{T_e - T_w} = \frac{u}{u_e}$$

would apply. The measured velocity and temperature profiles shown in this representation in Fig. 9 do indicate a linear relation between the normalized temperature and velocity profiles over most of the boundary layer, and thus imply that the eddy diffusivities are nearly equal there. However, in the wall region, where molecular transport becomes important, the temperature profile lies below the velocity profile because of the larger molecular diffusivity for heat than momentum transfer, i.e. $\alpha = [1/0{\cdot}7]\nu$.

V. SUMMARY AND CONCLUSIONS

Measured velocity and temperature profiles and friction coefficients were presented for a turbulent boundary layer in low-speed flow to appraise the effect of wall cooling on the mean structure of the boundary layer. The measurements were made in an air flow through the entrance region of a smooth, isothermal tube with negligible free-stream velocity variation,

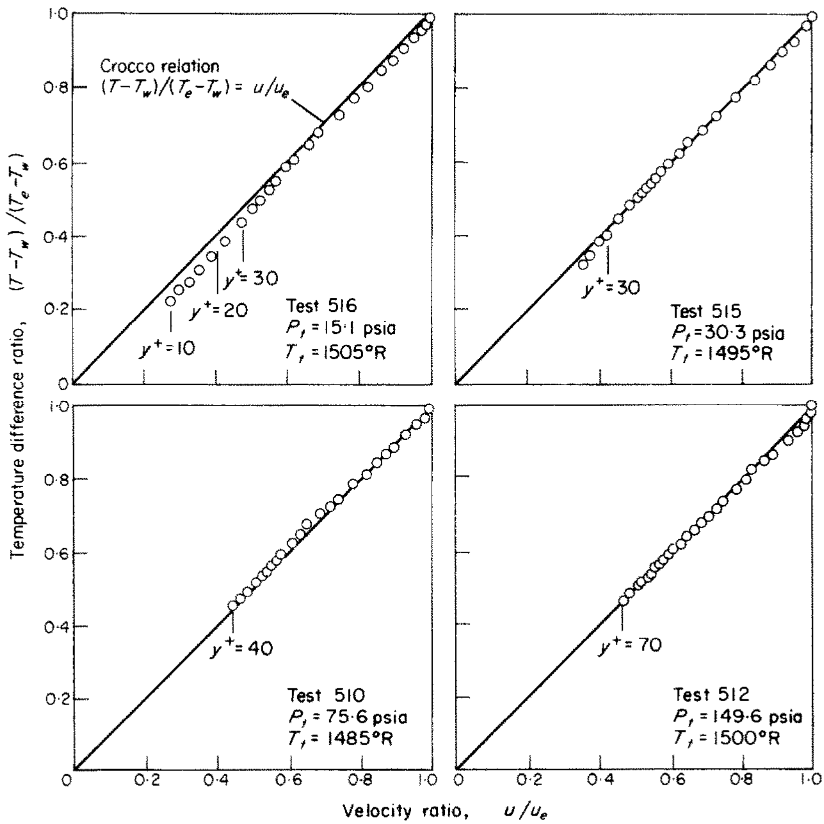

FIG. 9. Temperature vs. velocity profiles with cooling.

and spanned a range of momentum–thickness Reynolds numbers from 1500 to 36000 and wall-to-gas temperature ratios T_w/T_e down to 0·4. The effect of wall cooling i.e. decreasing T_w/T_e, was to increase the friction coefficient above the constant-property values; the increase amounted to about 20 per cent at $T_w/T_e = 0{\cdot}5$. Good agreement was found between the experimental results and three prediction methods; a reference temperature concept, Spalding and Chi's [1] empirical correlation, and Coles' [3] transformation theory in which an appropriate value of the viscosity-temperature exponent lies somewhere between 0·7 and 1·0.

The measured velocity and temperature profiles were found to lie above the constant-property profiles when viewed in terms of u^+, T^+ and y^+, with the amount of departure dependent on a cooling parameter β. Predicted velocity profiles from Prandtl's mixing-length theory and predicted velocity and temperature profiles from Coles' transformation theory were in fair agreement with the measured profiles. The measured velocity and temperature profiles indicated a wakelike behavior in the outer part of the layer.

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REFERENCES

1. D. B. Spalding and S. W. Chi, The drag of a compressible turbulent boundary layer on a smooth flat plate with and without heat transfer, *J. Fluid Mech.* **18**, 117–143 (1964).
2. J. H. Preston, The determination of turbulent skin friction by means of Pitot tubes, *Jl R. Aeronaut. Soc.* **58**, 109–121 (1954).
3. D. Coles, The turbulent boundary layer in a compressible fluid, *Physics Fluids* 7(9), 1403–1423 (1964) (also see Report No. P-2417, The Rand Corporation, Santa Monica, Calif., 1961).
4. V. C. Patel, Calibration of the Preston tube and limitations on its use in pressure gradients, *J. Fluid Mech.* **23**, Part 1, 185–208 (1965).
5. D. Coles, The law of the wake in the turbulent boundary layer, *J. Fluid Mech.* **1**, 191–226 (1956).
6. D. B. Spalding, A single formula for the law of the wall, *J. Appl. Mech.* Ser. E, 455–458 (1961).
7. L. H. Back, R. F. Cuffel and P. F. Massier, Laminar, transition and turbulent boundary layer heat-transfer measurements with wall cooling in turbulent air flow through a tube. *J. Heat Transfer* **91C**, 477–487 (1969).
8. D. G. Elliott, D. R. Bartz and S. Silver, Calculation of turbulent boundary-layer growth and heat transfer in axi-symmetric nozzles, Jet Propulsion Laboratory, Pasadena, Calif., TR 32-387 (Feb. 1963).
9. W. C. Reynolds, W. M. Kays and S. J. Kline, Heat transfer in the turbulent incompressible boundary layer with constant wall temperature, Part I. NASA memo 12-1-58 w (1958).
10. H. Schlichting, *Boundary Layer Theory*, 6th edn. McGraw-Hill, New York (1968).
11. S. W. Chi and D. B. Spalding, Influence of temperature ratio on heat transfer to a flat plate through a turbulent boundary layer in air, *Proc. 3rd Int. Heat Transfer Conf.*, Vol. II, pp. 41–49. AICHE (1966).
12. L. H. Back, Effects of severe surface cooling and heating on the structure of low-speed, laminar boundary gas flows with constant free-stream velocity, Jet Propulsion Laboratory, Pasadena, Calif., TR 32-1301 (1968).
13. S. V. Patankar, Calculation of compressible turbulent boundary layer on a flat plate, Imperial College of Science and Technology, Mech. Engng Dept. Report TWF/TN/28 (Aug. 1967).
14. J. E. Lewis, Compressible boundary layer and its low-speed equivalent, *AIAA Jl* **6**, 1185–1187 (1968).
15. R. G. Deissler, Turbulent heat transfer and friction in the entrance regions of smooth passages, *Trans. ASME* 77, 1221–1233 (1955).
16. E. R. Van Driest, Turbulent boundary layer in compressible fluids, *J. Aeronaut. Sci.* **18**(3), 145–160 (1951).
17. L. Crocco, Transformations of the compressible turbulent boundary layer with heat exchange, *AIAA Jl* **1**, 2723–2731 (1963).

APPENDIX

Friction Coefficients With Cooling

Spalding and Chi [1] *empirical method*

By comparison with numerous experimental data then available for supersonic flows over flat plates, a calculation procedure was proposed by which the friction coefficient when multiplied by a function F_c was postulated to be a function of the momentum-thickness Reynolds number multiplied by another function $F_{R\theta}$, i.e.

$$\frac{c_f}{2} F_c(Re_\theta F_{R\theta}).$$

The functions F_c and $F_{R\theta}$ that were assumed to depend upon Mach number and T_w/T_e are given in tabular form by Spalding and Chi [1]; F_c was obtained from mixing-length theory and $F_{R\theta}$ was determined empirically so that the prediction yielded the lowest root mean square error when compared to experimental data. Since the friction relation shown in Fig. 2 was established by Spalding and Chi for a constant-property low-speed flow denoted by barred quantities, i.e.

$$\frac{\bar{c}_f}{2}(\bar{R}e_\theta)$$

with $\bar{F}_c = 1$ and $\bar{F}_{R\theta} = 1$, there is a direct correspondence between the variable-property flow and the constant-property low-speed flow. These relationships are given by

$$\frac{c_f}{2}F_c = \frac{\bar{c}_f}{2}$$

$$Re_\theta F_{R\theta} = \bar{R}e_\theta$$

which can be combined to give

$$\frac{c_f}{2}Re_\theta F_c F_{R\theta} = \frac{\bar{c}_f}{2}\bar{R}e_\theta. \tag{A.1}$$

Spalding and Chi also tabulate the functions F_c and $F_{R\theta}$ in the low-speed limit, i.e. $M_e \to 0$, a condition for which the functions depend only upon T_w/T_e. However, their correlation was determined solely from supersonic flow data, there apparently being no low-speed-flow measurements available at that time.

Coles' transformation theory [3]

In the transformation, which is not restricted to turbulent boundary layers but applies to laminar boundary layers as well, three scaling functions, σ, ξ and η, determine the transformation of the stream function ψ and coordinates x and y, respectively; these relations are given by

$$\frac{\bar{\psi}}{\psi} = \sigma(x); \quad \frac{d\bar{x}}{dx} = \xi(x); \quad \frac{\bar{\rho}}{\rho}\frac{\partial\bar{y}}{\partial y} = \eta(x). \tag{A.2}$$

The friction coefficients and momentum-thickness Reynolds numbers in the two flows are related as follows for a perfect gas

$$\frac{c_f}{2}\frac{T_w}{T_e}\frac{[\bar{\mu}/\sigma]}{\mu_w} = \frac{\bar{c}_f}{2}, \qquad Re_\theta\frac{\mu_e}{[\bar{\mu}/\sigma]} = \bar{R}e_\theta. \tag{A.3}$$

Combining these equations gives the following relationship that is independent of the value of the scaling functions, which Coles refers to as the "law" of corresponding stations

$$\frac{c_f}{2}Re_\theta\frac{T_w}{T_e}\frac{\mu_e}{\mu_w} = \frac{\bar{c}_f}{2}\bar{R}e_\theta. \tag{A.4}$$

For laminar boundary layers with constant free-stream velocity the transformation requires that the scaling functions σ, ν and η be constant and that $\rho\mu$ = const. across the boundary layer. This latter condition is the well-known requirement that for a perfect gas, the viscosity must be proportional to temperature.

For turbulent boundary layers with constant free-stream velocity, Coles has established two independent relationships for the three scaling functions

$$\frac{\sigma}{\eta} = \text{const} = \frac{\bar{u}_e}{u_e}$$

$$\frac{\xi}{\eta} = \frac{\rho_w\mu_w}{\bar{\rho}\bar{\mu}}\frac{d}{d\theta}[\sigma\theta].$$

Coles claims these do not imply any restrictions on the equation of state, energy equation or viscosity relation. The third relationship, which Coles refers to as the sublayer hypothesis, is associated with the connection of the ratio $\bar{\mu}/\sigma$ with μ_s, i.e.

$$\mu_s = \frac{\bar{\mu}}{\sigma} \tag{A.5}$$

where μ_s is the viscosity evaluated at a mean sublayer temperature T_s. This interpretation followed by assuming the sublayer Reynolds number to be unaffected by compressibility or heat transfer and thus to be the same in the two flows

$$Re_s = \bar{u}_s^+\bar{y}_s^+ = \frac{\bar{\rho}\bar{u}_s\bar{y}_s}{\bar{\mu}} = \frac{u_s}{[\bar{\mu}/\sigma]}\int_0^{y_s}\rho\,dy = \frac{\rho_s u_s y_s}{[\bar{\mu}/\sigma]} = \text{const}$$

where ρ_s is a mean sublayer density. The friction relation given by equation (A.3) with $\bar{\mu}/\sigma = \mu_s$ is then specified once the mean sublayer temperature T_s is calculated. The form of the viscosity relation is important in this formulation. Coles has empirically evaluated the mean sublayer temperature defined by

$$T_s = \frac{1}{\bar{y}_s}\int_0^{\bar{y}_s}T\,d\bar{y} \tag{A.6}$$

from wall-friction measurements in turbulent boundary layers in supersonic air flows over adiabatic flat plates.

EFFET DU REFROIDISSEMENT DE LA PAROI SUR LA STRUCTURE MOYENNE D'UNE COUCHE LIMITE TURBULENTE DANS UN ÉCOULEMENT GAZEUX À FAIBLE VITESSE

Résumé—L'influence du refroidissement de la paroi sur la structure moyenne d'une couche limite turbulente dans un écoulement gazeux à faible vitesse, est discutée en fonction des profils mesurés de vitesse et de température et des coefficients de frottement, et l'on a fait des comparaisons avec les analyses semi-empiriques existantes des couches limites turbulentes. Les mesures ont été faites dans un écoulement d'air à travers la région d'entrée d'un tube lisse isotherme dans lequel la variation de la vitesse de l'écoulement libre était négligeable. Un accord satisfaisant a été obtenu entre la grandeur de l'augmentation du coefficient de frottement avec le refroidissement et les valeurs prédites à partir, 1) d'un concept de température de référence, 2) de la corrélation empirique de Spalding et Chi, et 3) de la théorie de la transformation de Coles dans laquelle une valeur appropriée de l'exposant de la variation de la viscosité la température se trouve entre 0,7 et 1,0. Les profils mesurés de vitesse et de température, lorsqu'ils sont représentés en fonction de u^+, T^+ et y^+, dépendaient d'un paramètre de refroidissement, indiqué par la théorie. Un bon accord a été obtenu entre les profils mesurés et prédits en utilisant les théories de la longueur de mélange de Prandtl et de la transformation de Coles.

AUSWIRKUNGEN DER WANDKÜHLUNG AUF DIE STRUKTUR EINER TURBULENTEN GRENZSCHICHT IN EINER LANGSAMEN GASSTRÖMUNG

Zusammenfassung—Der Einfluss der Wandkühlung auf die Struktur einer turbulenten Grenzschicht in einer langsamen Gasströmung wurde untersucht in Abhängigkeit von gemessenen Geschwindigkeits- und Temperatur-Profilen und Reibungskoeffizienten. Es wurden Vergleiche gemacht mit halb-empirischen Methoden für turbulente Grenzschichten. Die Messungen wurden in einer Lustströmung durch die Eintrittszone eines glatten, isothermen Rohres durchgeführt, wobei die Veränderung der Freistromgeschwindigkeit vernachlässigbar war. Es wurde zufriendenstellende Übereinstimmung festgestellt zwischen dem Betrag des Anwachsens des Reibungskoeffizienten mit der Kühlung und den vorhergesagten Werten von (1) einem Bezugstemperatur-Konzept, (2) Spalding and Chi's empirischer Beziehung, und (3) Cole's Transformations-Theorie, bei der ein passender Wert für den Exponenten der Viskositäts-Temperatur-Abhängigkeit zwischen 0,7 und 1,0 liegt.

Die gemessenen Geschwindigkeits- und Temperatur-Profile in der Darstellung bezogen auf u^+, T^+ und y^+ hängen von einem Kühlungs-Parameter ab, wie die Theorie angibt. Schlechtere Übereinstimmung herrscht zwischen den gemessenen Profilen und denen, die mittels des Mischungswegansatzes nach Prandtl und der Transformations-Theorie nach Colls bestimmt wurden.

ВЛИЯНИЕ ОХЛАЖДЕНИЯ СТЕНКИ НА ОСРЕДНЕННЫЕ ХАРАКТЕРИСТИКИ ТУРБУЛЕНТНОГО ПОГРАНИЧНОГО СЛОЯ ПРИ ТЕЧЕНИИ ГАЗА С МАЛОЙ СКОРОСТЬЮ

Аннотация—С помощью измеренных профилей скорости и температуры и коэффициентов трения рассматривается влияние охлаждения стенки на осредненные характеристики турбулентного пограничного слоя при течении газа с малой скоростью. Проведено сравнение с существующими полуэмпирическими теориями турбулентных пограничных слоев. Измерения проводились при течении воздуха через входную область гладкой изотермической трубы, в которой изменение скорости свободного потока было пренебрежимо малым. Найдено удовлетворительное согласование между увеличением коэффициента трения при охлаждении и значениями, рассчитанными по: 1) исходному значению температуры, 2) эмпирической корреляции Сполдинга и Ши и 3) теории преобразования Коулза, в которых соответствующие значения показателя степени вязкости и температуры находились в пределах 0,7 и 1. Измеренные профили скорости и температуры, выраженные через u^+, T^+ и y^+, зависели от параметра охлаждения. Найдено хорошее согласование между измеренными и рассчитанными профилями, включающими длину пути смешения Прандтля и преобразование Коулза.